Integrated Thermal Structural Analysis of Spacecraft Structures

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Abstract— **Finite element analysis procedures for predicting temperature response and associated thermal stress and buckling to 2D thermal structural analysis of advanced composite plates and shells are presented. Thermal analyses of structures are usually performed with non-consistent tools leading to an excessive effort in data adaptation. The application of 2D finite elements for this task relaxes this deficiency. With thermal composite theories, the temperature profile is assumed linear or quadratic in the thickness direction, but it is calculated by solving the Fourier's heat conduction equation. Finite element program has been developed using Semiloof shell elements and same eight noded isoparametric element concept for steady-state heat transfer. Results are verified for heat transfer, thermal stress and buckling analysis. New results are presented in terms of temperature, thermal stress and buckling for advanced composite plates. The results from this paper are to be useful mainly in nuclear reactor vessels and Thermal Protection System (TPS) in defence application.**

Keywords— **Thermal structural analysis; Composite plates and shells; Semiloof shell element; Temperature; Thermal stress**

I. Introduction

In our country, for the application of defence purpose, spacecraft structures play a vital role. A re-entry vehicle encounters enormous aerodynamic heating, the rate of which depends on the type of mission. Solid rocket motors are the main source of power for rockets. Many of the operational satellite launch vehicles and missiles around the world depend on solid motors for their propulsion during the initial phase of flight. The testing methodologies being adopted at ISRO for qualifying these materials from the structural integrity point of view are listed [1]. As the temperature increases, the TPS material begins to decompose. Thus, the design of suitable thermal protection system (TPS) becomes necessary for their successful operation. Noack et al., [2] used 4-noded shell elements to find thermal stress with a layer-wise theory for heat conduction problem using hybrid structures. In rapid acceleration of aircraft to a very high speed, friction in the boundary layer may raise the temperature of the surface of the aircraft with sufficient rapidity that large temperature differential may occur in the structure before interior positions heat up. This may result in thermal stresses of considerable magnitude [3].

The heat-transfer into the surface of the aircraft may be treated in a manner similar to conductive heat-transfer in low-speed flow. An appropriate heat-transfer coefficient is used in conjunction with the difference between the temperature at the solid surface and so-called "adiabatic wall temperature" or the temperature which would be attained in air at the solid surface if the surface was insulated. The heat-transfer coefficient and adiabatic wall temperature depend on the properties of the ambient air, the velocity of flow outside the boundary layer. During the initial phase of the trajectory, the aerodynamic heat generated on the body goes primarily to increase the body temperature, that is, the body behaves like a heat sink. Thus, the concept of boundary layer is very useful in heat-transfer problems [4] associated with aerodynamic heating. Yangjian et al., [5] studied analytically about convective heat transfer and thermal stress using FGM and composite materials for the plate.

In the last few decades, various TPS materials have been evolved. But, usage of the composite in all fields have been increased drastically now-a-days, because of their less weight and high stiffness property. Rolfes and Rohwer [6] analysed 2D finite elements for both laminated plates and cylindrical shells. So far, a lot of researches have been carried out about mechanical and thermo-mechanical behaviour of composite laminates [7] while very few works are available about integrated thermal structural analysis. Among various materials, Carbon-carbon composites, which are ceramic composites can withstand load beyond 2000°C. And carbon fibre reinforced polyimides have recently been used on radomes and fins operating at high temperatures for short and long duration because polyimides have high-temperature strength retention properties compared to epoxies and phenolics. Carbon-carbon composites have been successfully employed in the brake discs of aircraft, rocket nozzles and several other components operating in extreme thermal environments.

Kayhani et al [8] present an exact general solution for steady-state conductive heat transfer in cylindrical composite laminates. Bouazza and Amara [9] analysed three-dimensional thermo-elastic analysis of laminated plates subjected to uniform mechanical and thermal loads. Thus, deflections and stress resultants are obtained for symmetric and anti-symmetric cross-ply laminates for simply-supported boundary conditions at different temperatures. Closed-form formulations of a 2D higher-order shear deformation theory are presented by Khare [10] for the analysis of simply supported composite and sandwich laminated doubly curved shells under thermo-mechanical loading conditions. Sen [11] studied about elastoplastic thermal stresses in a thermoplastic composite disc that is reinforced by steel fibres, curvilinearly.

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Zhang and Yang [12] reviewed the recent development of the finite element analysis for laminated composite plates and summarised the future research areas. Plates and shells with different lamination and boundary conditions are analysed using high-order theories by Cinefra et al., [13].

The present work is concerned with the study of steady-state heat transfer for conduction problems in high-speed aerospace structures made of isotropic material using semiloof shell element. Semiloof shell element was developed by Irons [14] utilising isoparametric concept and isoparametric shell theory based on eight-noded element. In this work same isoparametric element is used to determine nodal temperature distribution. The semiloof element is extended to thermal stress analysis of laminated composite plates and shells by Thangaratnam et al [15]. Laminates with different fibre orientation have been considered in this analysis and the results are compared with the isotropic plate. The program developed COMSAP has been validated by comparing with the available results. The results are presented in terms of temperature, displacement and thermal stress.

II. Finite Element Formulation

In thermal analysis, the method of weighted residuals is frequently employed starting from the governing differential equations.

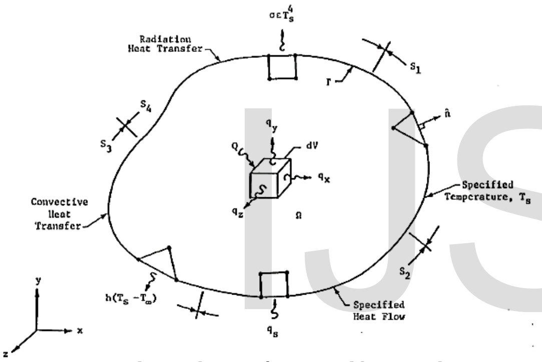

Fig. 1 3D solution domain for general heat conduction.

Fig. 1 shows several types of boundary conditions frequently encountered in the analysis. These boundary conditions are [16],

specified surface temperatures, T = Ts on S_1 (1)

surface heating, $q_x n_x + q_y n_y + q_z n_z = -q_s$ on S_2 (2)

surface convection, $q_x n_x + q_y n_y + q_z n_z = h(Ts - T_\infty)$ on S_3 (3)

surface radiation $q_x n_x + q_y n_y + q_z n_z = \varepsilon\sigma(T_s^4 - T_\infty^{-4}) - \propto q_r$ on S_4 (4)

where T, is the specified surface temperature; n_x, n_y, n_z are the direction cosines of the outward normal to the surface, q_s is the surface heating rate unit area, h is the convection coefficient, T_∞ is the convective medium temperature, σ is the Stefan-Boltzmann constant, $\varepsilon$is the surface emissivity, $\propto$is the surface absorptivity, and q_r is the incident radiant heat flow rate per unit area.

Semiloof shell element which has all the advantages of isoparametric formulation and which uses isoparametric shell theory and Discrete Kirchhoff Theory can overcome the locking phenomenon was developed by Irons [14] is used.

The governing differential equation for a 2D heat conduction problem is given by:

$$[C]\{\dot{T}\} + [[K_c] + [K_h]]\,\{T\} = \{Q_Q\} + \{Q_q\}\{Q_h\} \quad (5)$$

Where, [C]is the element capacitance matrix; [Kc] and $[K_h]$ are element conductance matrices corresponding to conduction and convection respectively. $[Q_Q]$, $[Q_h]$ and $[Q_q]$ are load vector due to internal heat generation, convection and specified surface heating.

The element matrices for analysing heat transfer problems are summarized herein.

$$[C] = \int_{-1}^{1}\int_{-1}^{1} \rho c\ [N(\varepsilon,\eta)]^T[N(\varepsilon,\eta)]\ t\ |J| d\,\varepsilon\, d\eta \quad (6)$$

$$[K_c] = \int_{-1}^{1}\int_{-1}^{1} [B(\varepsilon,\eta)]^T\ [k][B(\varepsilon,\eta)] t\ |J| d\,\varepsilon\, d\eta \quad (7)$$

$$[K_h] = \int_{-1}^{1}\int_{-1}^{1} h\,[N(\varepsilon,\eta)]^T[N(\varepsilon,\eta)]|J| d\,\varepsilon\, d\eta \quad (8)$$

$$[Q_Q] = \int_{-1}^{1}\int_{-1}^{1} Q\,[N(\varepsilon,\eta)]^T t\ |J| d\,\varepsilon\, d\eta \quad (9)$$

$$[Q_q] = \int_{-1}^{1}\int_{-1}^{1} q_s\,[N(\varepsilon,\eta)]^T |J| d\,\varepsilon\, d\eta \quad (10)$$

$$[Q_h] = \int_{-1}^{1}\int_{-1}^{1} h\,T_\infty\,[N(\varepsilon,\eta)]^T |J| d\,\varepsilon\, d\eta \quad (11)$$

Where, ρ is mass density,$[N(\varepsilon,\eta)]$ is shape function for 8 noded isoparametric element, t is thickness,|J| is determinant of jacobian,

$$[B(\varepsilon,\eta)] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial \varepsilon} & \frac{\partial N_2}{\partial \varepsilon} & \cdots & \frac{\partial N_8}{\partial \varepsilon} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_8}{\partial \eta} \end{bmatrix} \quad (12)$$

The conductivity matrix for the composite laminate$[k]$ is,

$$[k] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \quad (13)$$

The x and y axis conductivities k_x and $k_y$are related to the longitudinal and lateral axis conductivities $k_{11}$and k_{22} [17] respectively through the relation proposed by Shen and Springer [18], expressed as,

$$k_x = k_{11}\frac{\sum_{k=1}^{n}\cos^2\theta_k.h_k}{h} + k_{22}\frac{\sum_{k=1}^{n}\sin^2\theta_k.h_k}{h} \quad (14)$$

$$k_y = k_{11}\frac{\sum_{k=1}^{n}\sin^2\theta_k.h_k}{h} + k_{22}\frac{\sum_{k=1}^{n}\cos^2\theta_k.h_k}{h} \quad (15)$$

In the process of integration, Gauss-Legendre integration formula is used as,

$$[C] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ \rho c\ \ [N(\varepsilon_i,\eta_i)]^T[N(\varepsilon_i,\eta_i)]\left|J\left(\varepsilon_i,\eta_i\right)\right| \quad (16)$$

$$[K_c] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ [B(\varepsilon_i,\eta_i)]^T\ \ [k][B(\varepsilon_i,\eta_i)] t\ |J(\varepsilon_i,\eta_i)| \quad (17)$$

$$[K_h] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ \ h\,[N(\varepsilon_i,\eta_i)]^T[N(\varepsilon_i,\eta_i)]|J(\varepsilon_i,\eta_i)| \quad (18)$$

$$[Q_Q] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ Q\,[N(\varepsilon_i,\eta_i)]^T\ t\left|J\left(\varepsilon_i,\eta_i\right)\right| \quad (19)$$

$$[Q_q] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ q_s\ \ [N(\varepsilon_i,\eta_i)]^T\left|J\left(\varepsilon_i,\eta_i\right)\right| \quad (20)$$

$$[Q_h] = \sum_{i=1}^{NG}\sum_{j=1}^{NG} W_i W_j\ h\,T_\infty\,[N(\varepsilon_i,\eta_i)]^T |J\left(\varepsilon_i,\eta_i\right) \quad (21)$$

Where,W_i, W_j are weights and ε_i, η_i are Gauss-point locations. Thus nodal temperature values can be found out from the equations (16) to (21).

Based on minimization of the potential energy, formulation

$$\Pi = \sum^m \int \tfrac{1}{2} \begin{bmatrix} e \\ k \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} e \\ k \end{bmatrix} da - \int [q]^T [d]^T [p] da - \int \begin{bmatrix} e \\ k \end{bmatrix}^T \begin{bmatrix} N^T \\ M^T \end{bmatrix} da \quad (22)$$

$$\begin{bmatrix} e \\ k \end{bmatrix} = [H]\,[q] \quad (23)$$

Where, [H] is the strain matrix.

Substituting Eq. (23) in Eq. (22).

$$\Pi = \sum^m \int \tfrac{1}{2} [q]^T [H]^T [E][H][q] da - \int [q]^T [d]^T [p] da - \int [q]^T [H]^T \begin{bmatrix} N^T \\ M^T \end{bmatrix} da \quad (24)$$

Where $[E] = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$ (24a)

Differentiating the total potential energy with respect to nodal displacement [q] and equating to zero gives,

$$\Pi = \sum^m \int \tfrac{1}{2} [H]^T [E][H][q] da - \int [d]^T [p] da - \int [H]^T \begin{bmatrix} N^T \\ M^T \end{bmatrix} da = 0 \quad (25)$$

$$[K_S][q] = [f_m] + [f_T] \quad (26)$$

$[f_m]$ –Consistent nodal force vector due to mechanical load.

$$[f_m] = \sum \int [d^T][p]\, da \quad (27)$$

$[f_T]$–Consistent Thermal load vector.

$$[f_T] = \int [H]^T \begin{bmatrix} N^T \\ M^T \end{bmatrix} da \quad (28)$$

$[K_S]$ –Structural Stiffness Matrix.

$$[K_S] = \sum \int [H]^T [E][H] da \quad (29)$$

III. VERIFICATION

The existing program COMSAP [15] is extended for integrated thermal structural analysis to determine temperature distribution over the surface, displacement and resultant moment.

An isotropic homogenous square plate of size (a) = 100cm and thickness (h) = 1cm subjected to linear thermal gradient across the thickness and uniform over the surface has been analysed. All the edges are simply supported. The quarter plate is modelled using 8-noded quadrilateral isoparametric semiloof shell element with 4x4 meshes. Normal displacement (W) and moment resultants (M_{xx}/M_T), (M_{yy}/M_T) due to thermal loads in the plate along X-axis due to linear thermal gradient across thickness is compared as in Fig. 2 and 3 respectively. The material properties are: E= $2x10^6$ N/mm^2, μ= 0.3, α= $2x10^{-6}$strain/ ℃ , T_{top} = 100°Cand $T_{botttom}$=0°C, thermal moment resultant vector is given as M_T=EαTh^2/12.

From fig 2, it has been observed that the maximum value of displacement (W) is found at the centre of plate and gradually decreases to zero at the edge. Moment resultant along X-axis (M_{xx}) decreases gradually, whereas moment resultant along Y-axis (M_{yy}) increases gradually. Exact agreement is observed between Timoshenko (1961) [19] and the present result as shown in Fig. 2 and 3.

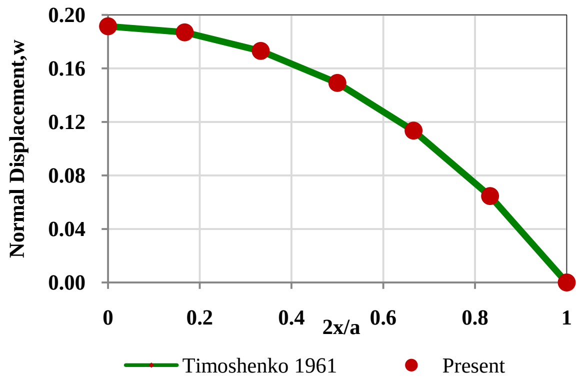

Fig.2 Normal displacement Vs thermal gradient

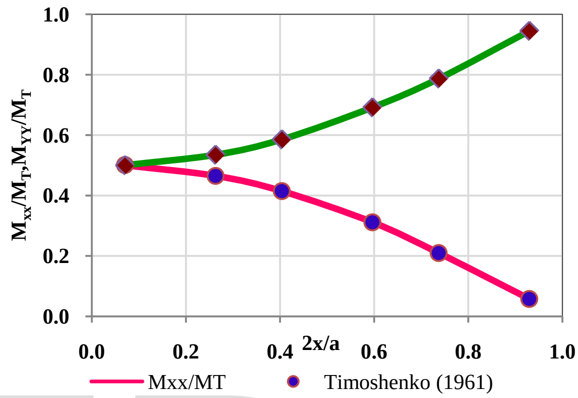

Fig.3 Moment resultants due to thermal loads

IV. RESULTS AND DISCUSSION

An isotropic square plate of size (a) = 100cm and thickness (h) = 1cm subjected to linear thermal gradient uniformly over the surface has been analysed. The material properties are: E= $2x10^6$ N/mm^2, μ= 0.3, α= $2x10^{-6}$strain/°C, Thermal conductivity coefficient in lateral direction k_{11} = 30, Thermal conductivity coefficient in transverse direction k_{22} = 30, Surrounding medium temperature T_∞ = 50°C, Internal heat generation [Q] = 10W/m^3.The isotropic rectangular plate is analysed for different aspect ratio such as a/b = 1, 1.5, 2, 2.5 and 3. The plate is simply supported on all the four sides, due to the symmetric condition quarter plate is modelled using 4×4 mesh.

Temperature distribution, displacement and thermal stress has been found out using the developed program. The maximum temperature distribution for the semiloof shell element along X and Y is displayed in Fig. 4 a) and b).

a) As the aspect ratio increases, the temperature also increases in both edges of x and y axis.
b) The maximum value of temperature is found at the centre of plate and at the edge T_∞ = 50°C is maintained.
c) Temperature decreases gradually from the centre of the plate to both the edges as shown in Fig.4 a) and b).

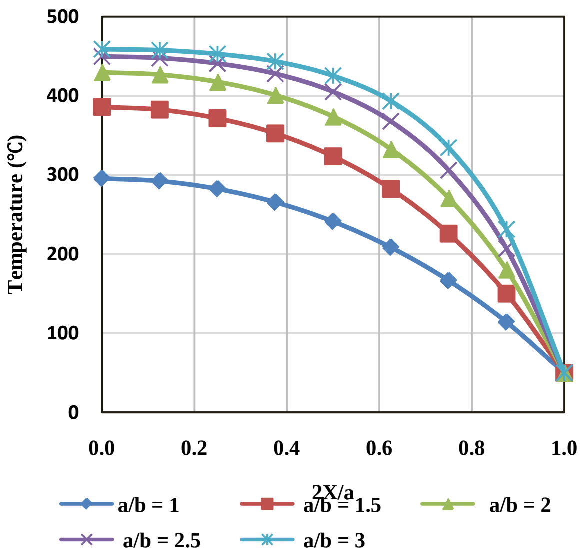

Fig 4. a)

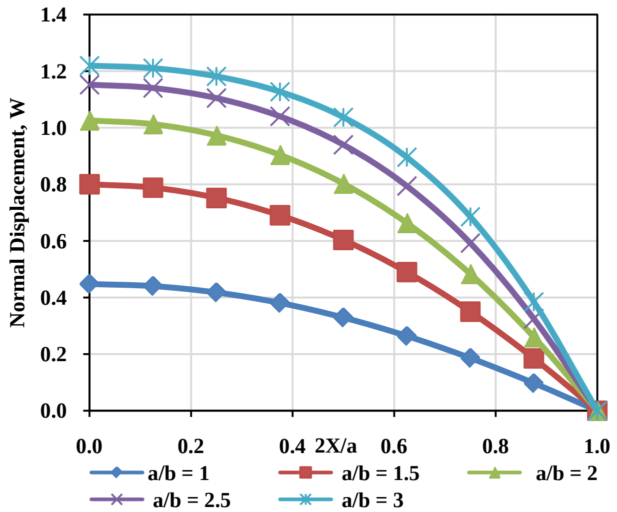

Fig 5. a)

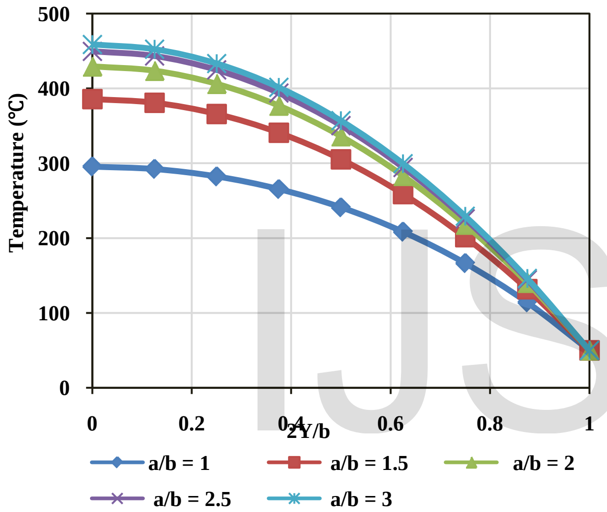

Fig 4. b)

Fig.4 Variation of temperature for different aspect ratio along a) X-axis b) Y-axis

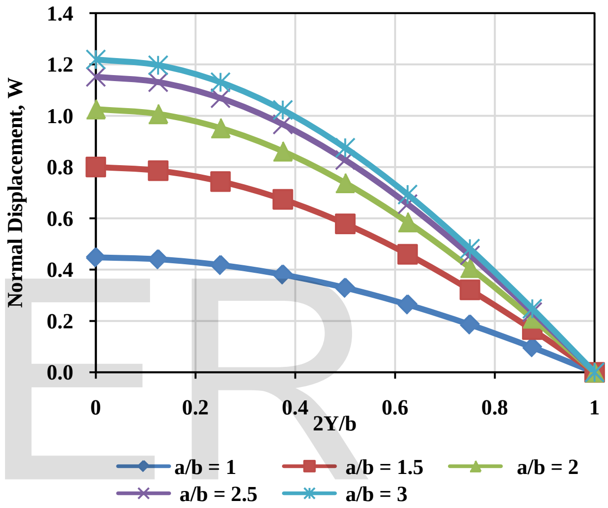

Fig 5. b)

Fig.5 Variation of normal displacement along a) X-axis b) Y-axis

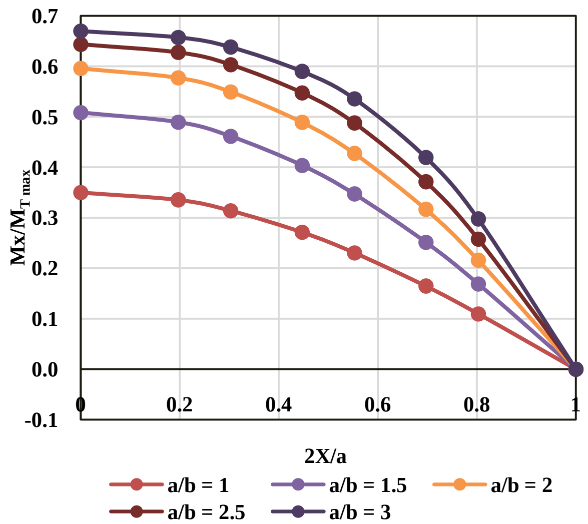

Fig 6. Moment resultant Mx along X-axis for different aspect ratio

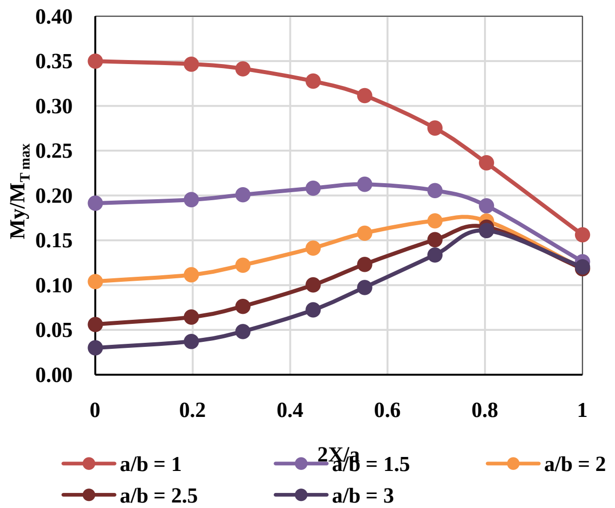

Fig.7 moment resultant My along X-axis for different aspect ratio

The maximum displacement along X and Y axis is displayed in Fig. 5 a) and b).

a) As the aspect ratio increases, the displacement also increases in both edges of x and y axis.
b) The maximum value of displacement is found at the centre of plate and gradually decreases to zero at the edge.

The resultant moment along X and Y is displayed in Fig. 6 and 7.

a) As the aspect ratio increases, the resultant moment M_x increases along x axis and M_y decreases along y axis.
b) M_{Tmax} represents the maximum thermal resultant in the isotropic plate.
c) The maximum value of moment is found at the centre of plate and gradually decreases to zero at the edge.

V. CONCLUSION

Integrated thermal structural analysis has been carried out using semiloof shell element and the program COMSAP is verified successfully. New results are obtained in terms of temperature, displacement and moment resultant. As the aspect ratio increases, temperature, displacement and resultant moment along X-axis also increases, except the resultant moment along Y-axis which decreases. The value of temperature, displacement and resultant moment is found maximum at the centre of the plate. Then, gradually decreases to zero at the edge of the isotropic plate.

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